Name: ANSWERS

Instructor: Bullwinkle

Math 10120, Exam 3.
November 18, 2014

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators **are** allowed
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(•)	(e)
2.	(a)	(b)	(•)	(d)	(e)
3.	(•)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(•)	(d)	(e)
5.	(a)	(•)	(c)	(d)	(e)
6.	(•)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(•)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(•)	(d)	(e)
10.	(a)	(b)	(c)	(•)	(e)

Please do NOT write in this box.

Multiple Choice	______
11.	______
12.	______
13.	______
15.	______
Total	______

Multiple Choice

1.(5pts) The daily high temperatures (in degrees Fahrenheit) one week in February were 20, 15, 19, 25, 24, 13, and 17. Find the median.

(a) 18.5 (b) 17 (c) 19.5

(d) 19 (e) 20

Solution. Order the data: 13, 15, 17, 19, 20, 24, 25. There are 7 data points and $\frac{7+1}{2} = 4$ so the median is obtained by counting 4 from either end: hence 19.

2.(5pts) On a 20 point quiz with each multiple choice problem worth 5 points, a class had the following scores

# Score	Frequency
0	3
5	4
10	10
15	25
20	8

The population mean is 13.1. Find the population standard deviation to two decimal places.

(a) 7.23 (b) 2.46 (c) 5.19 (d) 4.38 (e) 6.12

Solution.

# Score $= x_i$	Frequency $= f_i$	$x_i - \mu$	$(x_i - \mu)^2$
0	3	-13.1	171.61
5	4	-8.1	65.61
10	10	-3.1	9.61
15	25	1.9	3.61
20	8	6.9	47.61
$n = 50$	$\mu = \frac{\sum x_i f_i}{n} = \frac{1344.5}{50} = 13.1$		$\sigma^2 = \frac{\sum (x_i - \mu)^2 f_i}{n} = 26.89$

so $\sigma = \sqrt{26.89} \approx 5.185556865$.

3.(5pts) The probability distribution of the random variable X is shown in the accompanying table.

x	0	5	10	15	20
P$(X = x)$	0.21	0.16	0.18	0.21	0.24

Which statement (a)-(e) is correct?

(a) $P(X \geqslant 10) = 0.63$

(b) $P(X \geqslant 10) = 0.82$

(c) $P(X \geqslant 10) = 0.55$

(d) $P(X \geqslant 10) = 0.211$

(e) $P(X \geqslant 10) = 0.18$

Solution. Since $P(X \geqslant 10) = P(X = 10) + P(X = 15) + P(X = 20)$, $P(X \geqslant 10) = 0.18 + 0.21 + 0.24 = 0.63$.

4.(5pts) A sample of 4 elements x_1, x_2, x_3 and x_4 is taken from a population of $1,000,000$ elements. Which formula gives the sample standard deviation?

$\bar{x}$ denotes the sample mean below.

(a) $\sqrt{\dfrac{(x_1 - \bar{x})^2 + (x_2 - -\bar{x})^2 + (x_3 - -\bar{x})^2 + (x_4 - -\bar{x})^2}{4}}$

(b) $\sqrt{\dfrac{x_1 + x_2 + x_3 + x_4}{4}}$

(c) $\sqrt{\dfrac{(x_1 - -\bar{x})^2 + (x_2 - -\bar{x})^2 + (x_3 - -\bar{x})^2 + (x_4 - -\bar{x})^2}{3}}$

(d) $\sqrt{\dfrac{1 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 + 4 \cdot x_4}{4}}$

(e) $\sqrt{\dfrac{x_1 + x_2 + x_3 + x_4}{3}}$

Solution. Look up the correct formula in the book.

5.(5pts) A baseball player has a 0.250 batting average ($= \frac{\#\text{ hits}}{\#\text{ times at bat}}$). In the course of last season he came to bat 300 times. What is H, his expected number of hits for the season and what is the standard deviation D, for this player hits?

(a) $(H, D) = (225, 7.50)$ (b) $(H, D) = (75, 7.50)$ (c) $(H, D) = (75, 4.33)$

(d) $(H, D) = (7.5, 2.70)$ (e) $(H, D) = (100, 8.66)$

Solution. This is a binomial distribution problem with $n = 300$, $p = 0.25$ and $q = 1 - p = 0.75$. Hence $H = \mu = np = 300 \cdot 0.25 = 75$. Then $D = \sigma = \sqrt{npq} = \sqrt{\mu q} = \sqrt{75 \cdot 0.75} = 7.5$.

6.(5pts) If Z is a standard normal random variable, what is $P(-0.5 \leq Z \leq 1.5)$.

Note You will find tables for the standard normal distribution at the end of the exam.

(a) 0.6247 (b) 0.3085 (c) 0.2417

(d) 0.8413 (e) 0.9332

Solution. $P(-0.5 \leq Z \leq 1.5) = P(Z \leq 1.5) - P(Z \leq -0.5)$
$= A(1.5) - A(-0.5)$(from tables) $= .9332 - .3085 = .6247$.

7.(5pts) The height (at the shoulder) of adult snopalopagus' is normally distributed with mean $\mu = 10$ ft. and standard deviation $\sigma = 3$ ft. If I choose an adult snopalopagus at random from the population, what is the probability that it will have a shoulder height greater than 15.4 feet?

(a) 0.4591 (b) 0.9641 (c) 0.0359

(d) 0.0001 (e) 0.1358

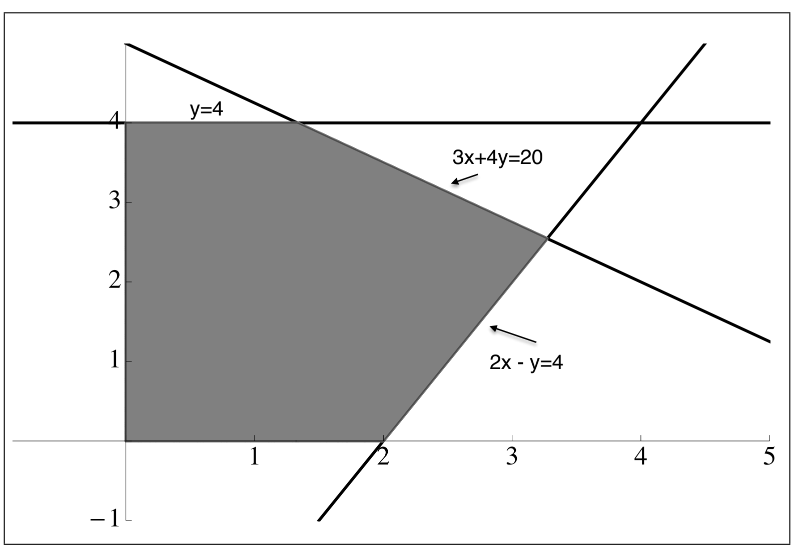
Solution.

$$P(X > 15.4) = P(Z > \frac{15.4 - 10}{3}) = P(Z > 1.8),$$

where Z is a standard normal random variable. Using the tables, we get:

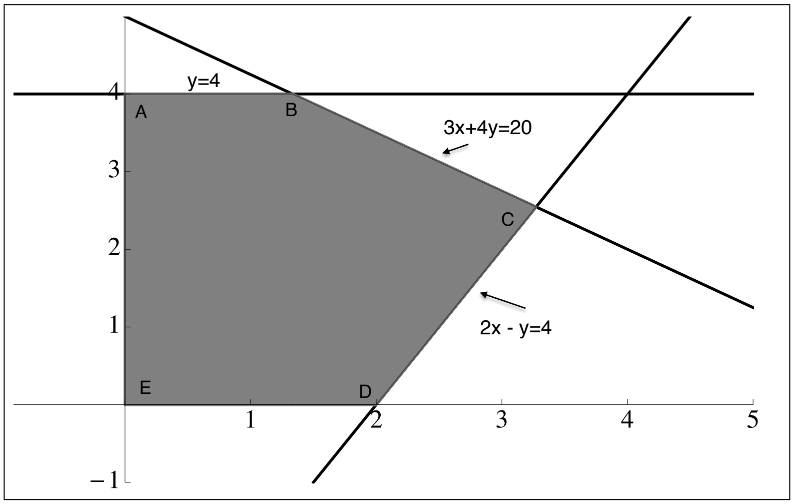
$$P(Z > 1.8) = 1 - P(Z \leq 1.8) = 1 - .9641 = .0359.$$

8.(5pts) What is the maximum of the objective function (rounded off to two decimal places) $2x + 4y$ on the feasible set shown as the shaded region in the diagram below?



(a) 18.67 (b) 18.18 (c) 22.48 (d) 13.33 (e) 16.73

Solution.



We label the vertices A, B, C, D and E as shown. The co-ordinates of E are $(0, 0)$,
the co-ordinates of A are $(0, 4)$ and
the co-ordinates of D are $(2, 0)$.
At B, $y = 4$ and $3x + 4y = 20$ which gives $3x + 16 = 20$ and $x = 4/3$. Thus the co-ordinates of B are $(4/3, 4)$.
At C, $2x - y = 4$ gives $y = 2x - 4$. Plugging this into the equation $3x + 4y = 20$ gives $3x + 4(2x - 4) = 20$ which gives $11x - 16 = 20$ or $11x = 36$. Thus $x = 36/11$. Plugging this into $y = 2x - 4$ gives $y = 28/11$. Therefore the co-ordinates of C are $(36/11, 28/11)$.

We check the value of the objective function on each vertex:

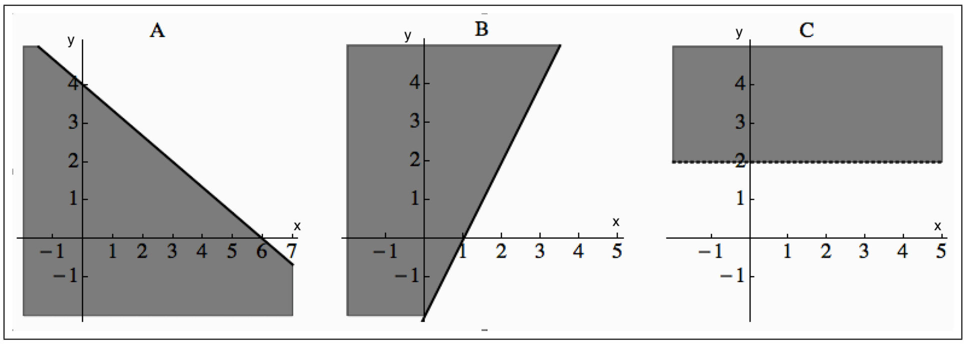
vertex	$2x + 4y$
$(0, 0)$	0
$(0, 4)$	16
$(2, 0)$	4
$(4/3, 4)$	18.67
$(36/11, 28/11)$	16.73

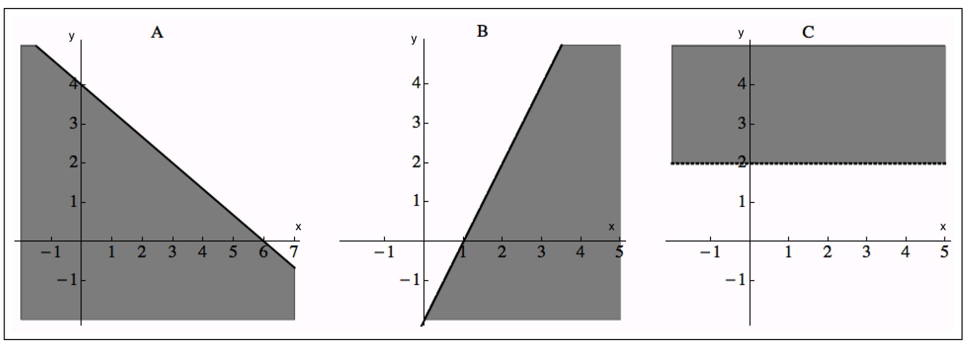
The maximum is 18.67.

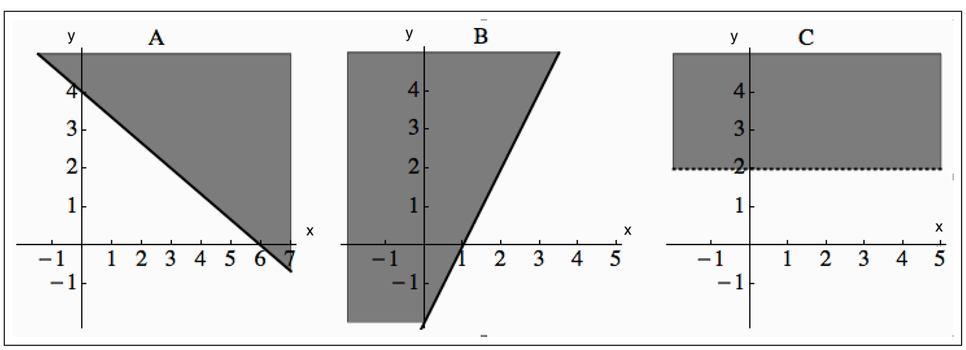
9.(5pts) Below we give three inequalities A, B and C:

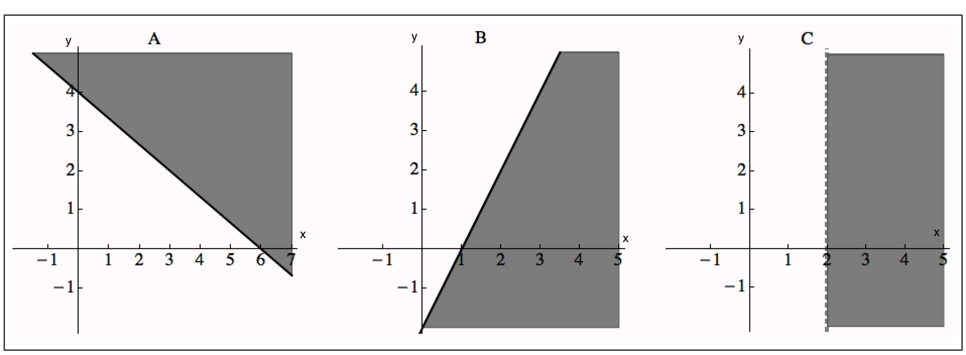
$$\begin{array}{lrcrcl} \text{A:} & 2x & + & 3y & \geq & 12 \\ \text{B:} & 2x & - & y & \leq & 2 \\ \text{C:} & & & y & > & 2 \end{array}$$

Which of the pictures below shows the correct graphs of A, B and C?

(a) 

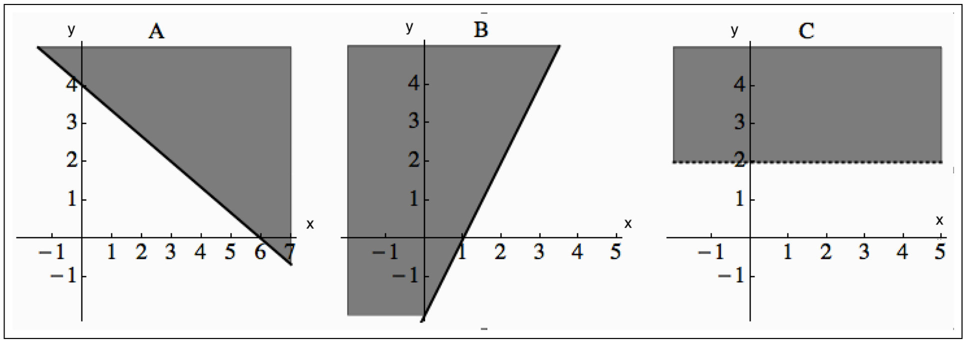
(b) 

(c) 

(d) 

(e) None of the above

Solution. For A the point $(0,0)$ is not in the solution set (and not on the corresponding line), so the solution set is above the line $2x+3y=12$. The graph includes the line.
For B the point $(0,0)$ is in the solution set (and not on the corresponding line), so the solution set is above the line $2x-y=2$. The graph includes the line.
The graph of the inequality C is the region above the horizontal line $y=2$, the graph does not include the line. Thus the solution is



10.(5pts) Joe and Mary run a small business producing tables and chairs. Joe produces the parts for the furniture and Mary assembles the furniture. It takes Joe 10 hours to produce the parts for a table and it takes him 4 hours to produce the parts for one chair. It takes Mary one hour to assemble a table and two hours to assemble a chair. Joe has 40 hours to devote to making furniture parts each week and Mary has 8 hours to devote to assembling furniture each week. Each table sold brings a profit of \$200 and each chair sold brings a profit of \$50. Mary and Joe sell all of the furniture that they produce and wish to maximize profits. Let x denote the number of tables that Mary and Joe make in a week and let y denote the number of chairs they make in a week, which of the following give the constraints on x and y and the objective function?

(a)
$$\begin{array}{rrcl} 10x + & y & \leq & 40 \\ 4x + & 2y & \leq & 8 \\ & y & \geq & 0 \\ & x & \geq & 0 \\ \text{Obj. Function:} & 200x & + & 50y \end{array}$$

(b)
$$\begin{array}{rrcl} 4x + & 10y & \leq & 40 \\ 2x + & y & \leq & 8 \\ & y & \geq & 0 \\ & x & \geq & 0 \\ \text{Obj. Function:} & 50x & + & 200y \end{array}$$

(c)
$$\begin{array}{rrcl} 10x + & 2y & \leq & 8 \\ x + & 4y & \leq & 40 \\ & y & \geq & 0 \\ & x & \geq & 0 \\ \text{Obj. Function:} & 200x & + & 50y \end{array}$$

(d)
$$\begin{array}{rrcl} 10x + & 4y & \leq & 40 \\ x + & 2y & \leq & 8 \\ & y & \geq & 0 \\ & x & \geq & 0 \\ \text{Obj. Function:} & 200x & + & 50y \end{array}$$

(e)
$$\begin{array}{rrcl} x + & 4y & \leq & 40 \\ 10x + & 2y & \leq & 8 \\ & y & \geq & 0 \\ & x & \geq & 0 \\ \text{Obj. Function:} & 50x & + & 200y \end{array}$$

Solution. Joe's time constraint translates to $10x + 4y \leq 40$
Mary's time constraint translates to $x + 2y \leq 8$.
In addition, we must have $x \geq 0$ and $y \geq 0$. Profit is given by $200x + 50y$.

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12pts) Joe took admissions tests for two different apprenticeship programs. On the first, where the mean was 74 and the standard deviation 8, he scored 88. On the second, with mean 59 and standard deviation 12, he scored 79.

(a) Compute Joe's z-score on the first test.

(b) Compute Joe's z-score on the second test.

(c) On which test did Joe do better?

Solution. The z-score is $\frac{x-\mu}{\sigma}$ so for the first exam we get $\frac{88-74}{8} = \frac{14}{8} = \frac{7}{4} = 1.75$. For the second exam we get $\frac{79-59}{12} = \frac{20}{12} = \frac{5}{3} = 1.2333\cdots$. Hence Joe did better on the first exam.

Initials: ____________

12.(12pts) Compute μ, $\sigma^2(X)$, and $\sigma(X)$ for the random variable defined as follows.

x_i	100	160	230
p_i	0.3	0.5	0.2

$\mu =$ ________, $\sigma^2(X) =$ __________, $\sigma(X) =$ _________.

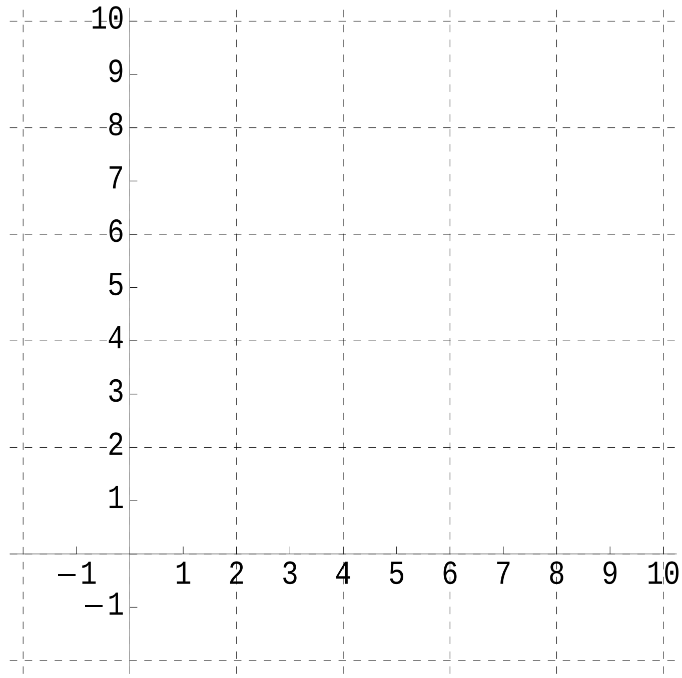
Solution.

x_i	p_i	$x_i - \mu$	$(x_i - \mu)^2$
100	0.3	-56	3136
160	0.5	4	16
230	0.2	74	5476
$n = 490$	$\mu = \sum x_i p_i = 156$		$\sigma^2 = \sum (x_i - \mu)^2 p_i = 2044$

Hence $\sigma = \sqrt{2044} \approx 45.2106182218$.

13.(12pts) (a) Graph the feasible set corresponding to the following set of inequalities on the set of axes provided. (Make sure you shade the region corresponding to the feasible set and clearly identify the region as the feasible set.)

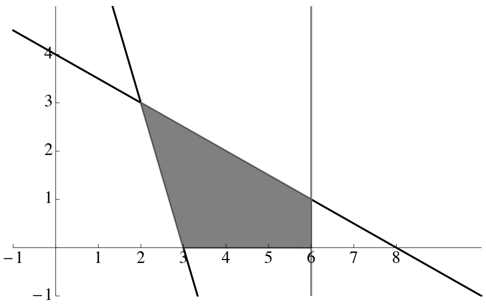
$$\begin{aligned} 2y + x &\le 8 \\ y + 3x &\ge 9 \\ x &\ge 0 \\ y &\ge 0 \\ x &\le 6 \end{aligned}$$



(b) Find the vertices of the above feasible set.

(c) Find the **minimum** value of the objective function $5x + 4y$ on the above feasible set.

Solution.



The co-ordinates of the vertex D are $(3, 0)$ and
the co-ordinates of the vertex C are $(6, 0)$.
At B, we have $x = 6$ and $2y + x = 8$, thus $2y = 2$ and $y = 1$. Therefore the co-ordinates at B are $(6, 1)$.
At A, we have $y = 9 - 3x$. Substituting this into $2y + x = 8$, we get $2(9 - 3x) + x = 8$ or $18-5x = 8$, giving us that $x = 2$. This gives us that $y = 9-3(2) = 3$. Thus the co-ordinates of A are $(2, 3)$.

We check the value of the objective function on each vertex:

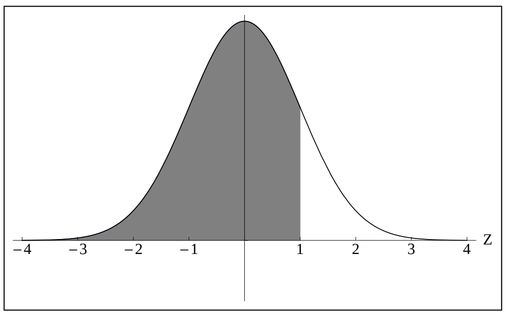
vertex	$5x + 4y$
$(3, 0)$	15
$(6, 0)$	30
$(6, 1)$	34
$(2, 3)$	22

The minimum is 15.

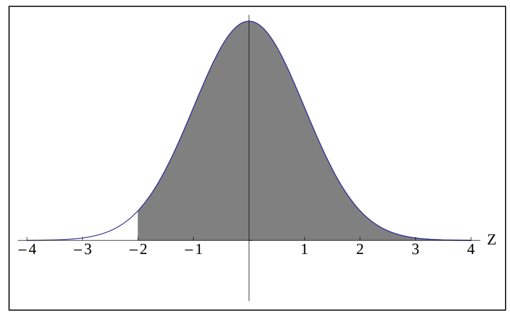
14.(12pts) The Lifetime of the Northern Basselope is normally distributed with mean 80 years and standard deviation 4 years. Let X denote the lifetime of a Northern Basselope chosen at random from the population.

(i) The graphs below represent areas under the standard normal distribution, with mean zero and standard deviation 1. Which of the shaded areas shown below equals $P(X \leq 72)$, where X is the random variable described in Part (a).

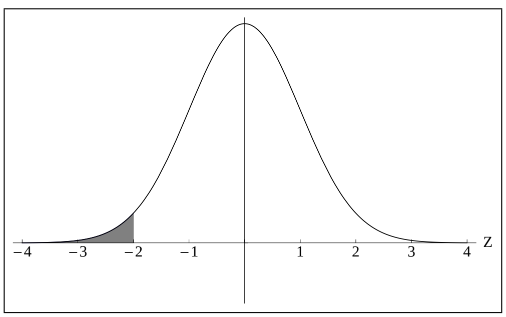
(a)



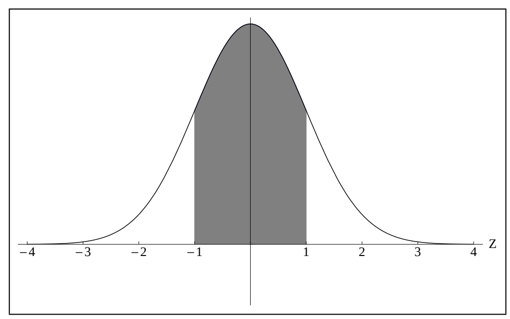
(b)



(c)



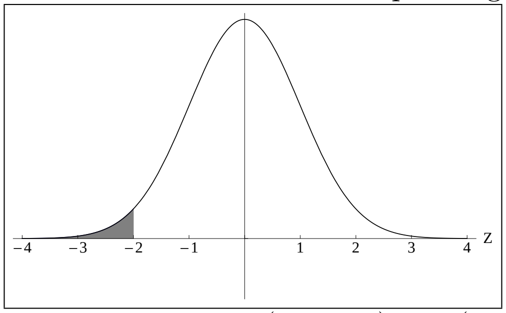
(d)



(ii) Calculate $P(X \leq 72)$, where X is the random variable described in Part (a).

(iii) In a random sample of 100 Northern Basselopes, how many would you expect to live to an age **greater than** 72 years?

Solution. $P(X \leq 72) = P(Z \leq \frac{72-80}{4} = P(Z \leq -2)$, where Z is a standard normal random variable. The corresponding picture is given by



From the tables, $P(X \leq 72) = P(Z \leq -2) = 0.0228$.
$P(X \geq 72) = P(Z \geq -2) = 1 - P(Z \leq -2) = 0.9821$. In a random sample of 100 Northern Basselopes, one would expect about 98 to live past 72 years.

15.(2pts) You will get this 2 points if your instructor can read your name easily on the front page of the exam and you mark the answer boxes with an X (as opposed to a circle or any other mark).

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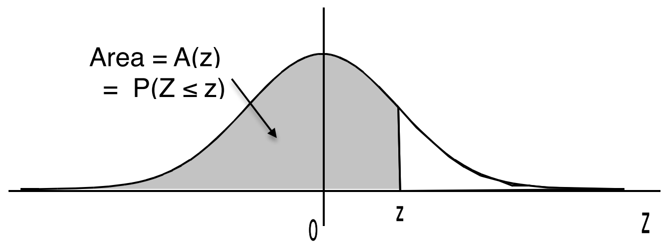
Initials: ____________

Areas under the Standard Normal Curve



z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$
−3.50	.0002	−2.00	.0228	−.50	.3085	1.00	.8413	2.50	.9938
−3.45	.0003	−1.95	.0256	−.45	.3264	1.05	.8531	2.55	.9946
−3.40	.0003	−1.90	.0287	−.40	.3446	1.10	.8643	2.60	.9953
−3.35	.0004	−1.85	.0322	−.35	.3632	1.15	.8749	2.65	.9960
−3.30	.0005	−1.80	.0359	−.30	.3821	1.20	.8849	2.70	.9965
−3.25	.0006	−1.75	.0401	−.25	.4013	1.25	.8944	2.75	.9970
−3.20	.0007	−1.70	.0446	−.20	.4207	1.30	.9032	2.80	.9974
−3.15	.0008	−1.65	.0495	−.15	.4404	1.35	.9115	2.85	.9978
−3.10	.0010	−1.60	.0548	−.10	.4602	1.40	.9192	2.90	.9981
−3.05	.0011	−1.55	.0606	−.05	.4801	1.45	.9265	2.95	.9984
−3.00	.0013	−1.50	.0668	.00	.5000	1.50	.9332	3.00	.9987
−2.95	.0016	−1.45	.0735	.05	.5199	1.55	.9394	3.05	.9989
−2.90	.0019	−1.40	.0808	.10	.5398	1.60	.9452	3.10	.9990
−2.85	.0022	−1.35	.0885	.15	.5596	1.65	.9505	3.15	.9992
−2.80	.0026	−1.30	.0968	.20	.5793	1.70	.9554	3.20	.9993
−2.75	.0030	−1.25	.1056	.25	.5987	1.75	.9599	3.25	.9994
−2.70	.0035	−1.20	.1151	.30	.6179	1.80	.9641	3.30	.9995
−2.65	.0040	−1.15	.1251	.35	.6368	1.85	.9678	3.35	.9996
−2.60	.0047	−1.10	.1357	.40	.6554	1.90	.9713	3.40	.9997
−2.55	.0054	−1.05	.1469	.45	.6736	1.95	.9744	3.45	.9997
−2.50	.0062	−1.00	.1587	.50	.6915	2.00	.9772	3.50	.9998
−2.45	.0071	−.95	.1711	.55	.7088	2.05	.9798		
−2.40	.0082	−.90	.1841	.60	.7257	2.10	.9821		
−2.35	.0094	−.85	.1977	.65	.7422	2.15	.9842		
−2.30	.0107	−.80	.2119	.70	.7580	2.20	.9861		
−2.25	.0122	−.75	.2266	.75	.7734	2.25	.9878		
−2.20	.0139	−.70	.2420	.80	.7881	2.30	.9893		
−2.15	.0158	−.65	.2578	.85	.8023	2.35	.9906		
−2.10	.0179	−.60	.2743	.90	.8159	2.40	.9918		
−2.05	.0202	−.55	.2912	.95	.8289	2.45	.9929		